J. Scattering by Phonons provides ways to probe $\omega(\vec{q})$

Picture:

$$u_{\vec{R}}(t) = \sum_{q} \Big(e^{i\vec{q}\cdot\vec{R}} \underbrace{e^{-i\omega(\vec{q})t}}_{[1]} \frac{A_{\vec{q}}}{\sqrt{N}} + e^{-i\vec{q}\cdot\vec{R}} \overbrace{e^{i\omega(\vec{q})t}}^{[2]} \frac{A^*_{\vec{q}}}{\sqrt{N}} \Big) \qquad (13)$$

$\underbrace{\text{Some incident light/or neutron}}$ (initial)

↓

outgoing light/or neutron (final)

[1] $\sim \underbrace{\int_0^t e^{i(E_{final}-E_{initial})t'/\hbar}\, e^{-i\omega(\vec{q})t'}\, dt'}$

requires $E_{final} = E_{initial} + \hbar\omega(\vec{q})$

higher energy out (E_{final}); comes in ($E_{initial}$) and absorbs a phonon ($\hbar\omega(\vec{q})$)

[$A_{\vec{q}} \rightarrow \hat{b}_{\vec{q}} \Rightarrow$ annihilates phonon]

Also, $\vec{k}_f = \vec{k}_i + \vec{q}$ $\quad (\hbar\vec{k}_f = \hbar\vec{k}_i + \hbar\vec{q})$

[2] gives $E_{final} = E_{initial} - \hbar\omega(\vec{q})$

lower energy out (E_{final}); comes in ($E_{initial}$) and creates a phonon ($\hbar\omega(\vec{q})$)

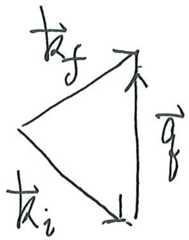
[$A^*_{\vec{q}} \rightarrow \hat{b}^{\dagger}_{\vec{q}} \Rightarrow$ creates phonon]

Also, $\vec{k}_f = \vec{k}_i - \vec{q}$ $\quad (\hbar\vec{k}_f = \hbar\vec{k}_i - \hbar\vec{q})$

Using "Light" [$\underbrace{\text{X-ray}}_{}$, optical range] or $\underbrace{\text{thermal neutrons}}_{}$

harder to sort out $\pm\hbar\omega(\vec{q})$ from $\hbar\omega_{x\text{-ray}}$ scale [10^5 bigger]

energy $\sim 0.08\,\mathrm{eV}$ (easy to sort out $\underbrace{\pm\hbar\omega(\vec{q})}_{\text{comparable scale}}$)



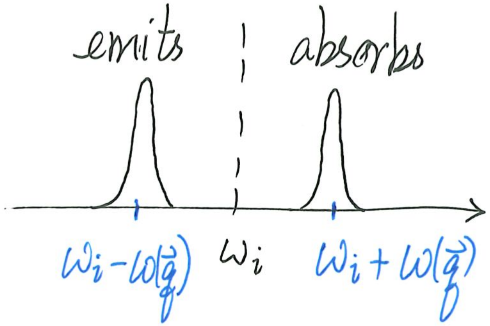
detector location

$$\vec{k}_f = \vec{k}_i + \underbrace{\vec{q}}_{}$$

annihilates $\hbar\vec{q}$ phonon

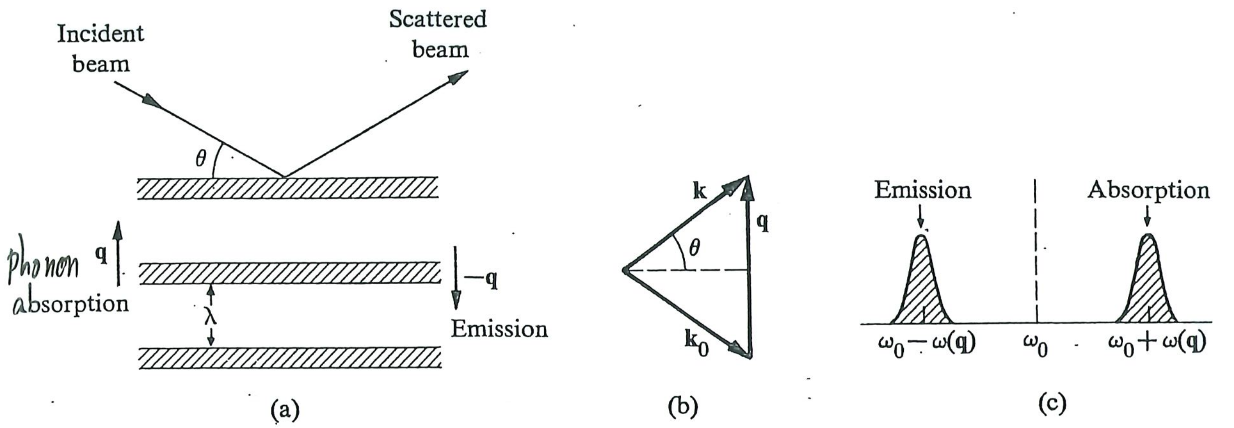
(creates $-\hbar\vec{q}$ phonon)

$$\hbar\omega_f = \hbar\omega_i \pm \hbar\omega(\vec{q})$$



This is how (inelastic) scattering gives $\underbrace{\omega(\vec{q})}_{\text{the dispersion relation}}$

illustrating lattice vibrations of specific q set up a new λ [repeating once for many layers of atoms] for scattering



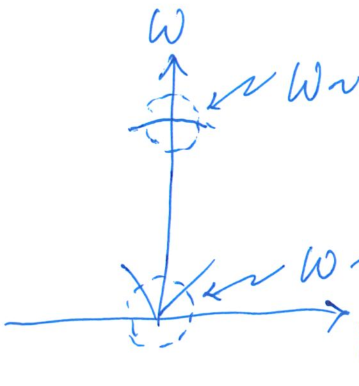
Scattering of x-rays by phonons. (a) The vibrating lattice acts as a set of planes at spacing equal to λ. Absorption of a phonon $\mathbf{q}$ and emission of a phonon $-\mathbf{q}$ lead to the same momentum conservation, and hence the two processes are observed simultaneously at the detector. Their frequencies are different, however. (b) Conservation of momentum for x-ray photon–phonon collision. (c) Shifted x-ray frequencies.

Visible Range

- Involving Acoustic phonons: Brillouin Scattering
- Involving Optical phonons: Raman Scattering

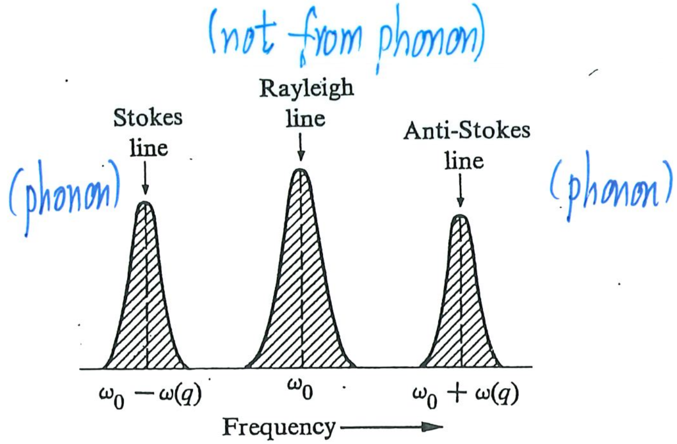
Visible range: $\lambda \sim 500\,nm$, $\underbrace{k \approx 10^5\,cm^{-1}}_{\text{expected } q \text{ (phonon) probed } \sim k} \ll \underbrace{10^8\,cm^{-1} \approx \frac{\pi}{a}}_{\text{B.Z. size}}$

$\Rightarrow$ probes a small part of B.Z. for $\omega(\vec{q})$

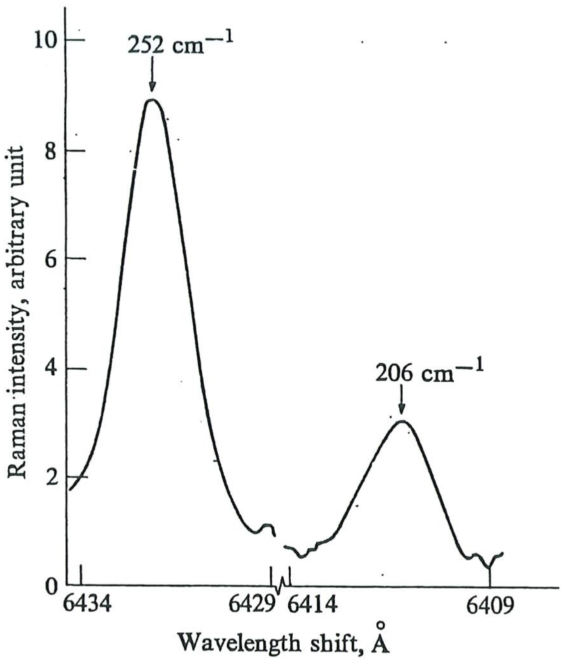


$\omega \sim 10^{13}\,s^{-1}$ (easier to see $\pm\omega(q)$)

$\omega \sim 10^{11}\,s^{-1}$ (harder to sort out the wings $\pm\omega(q)$)



Raman spectrum, showing undisplaced Rayleigh line, as well as Stokes and anti-Stokes lines.



Raman spectrum of ZnSe, showing scattering from both longitudinal (252 cm^{-1}) and transverse (206 cm^{-1}) optical phonons.

K. Phonon Scattering with Electron as Causes of resistance

Picture: $\sigma = \frac{ne^2}{m^*}\tau$

per volume

conductivity of "free electrons"

mass (effective)

τ = (average) time between successive collisions of electrons with something

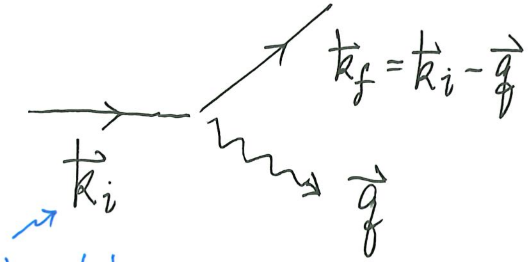
Most importance for semiconductors:

- $\boxed{\text{phonons}}$
- $\boxed{\text{impurities}}$
- defects
- sample boundaries

- $u_{\vec{R}}(t)$ (Eq. (13)) represents deviation of atom at $\vec{R}$ from equilibrium position

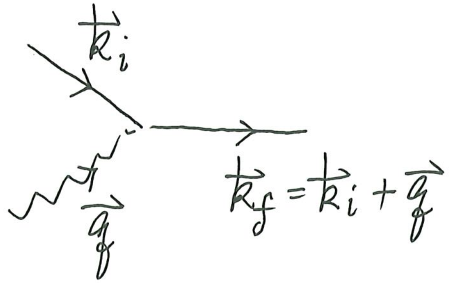
When atoms all at $\{\vec{R}_i\} \Rightarrow V_{periodic}(\vec{r})$ ← goes into TISE ⇓ Bloch states (bands)

∴ Electrons in Bloch states see deviations $\underbrace{u(t)}_{\text{as perturbations}}$

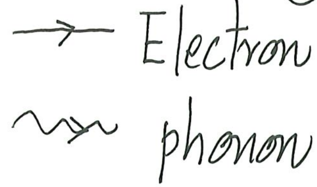


Bloch state (band n)

Emits a phonon



Absorbs a phonon



Each process contributes to give a finite τ

E.g. more phonons (higher temperature) $\Rightarrow$ shorter τ

$\therefore$ σ is smaller (more resistive)